Lecture 4
RF emittance in photocathode guns

The following derivation of the rf emittance comes from K-J. Kim, NIM A275(1989)201-218.

The beam emittance from an rf gun can be separated into the thermal emittance (as discussed in Section II), the rf emittance, the space-charge emittance and the coupling of the rf and space charge emittances which can be combined as root-mean-squares to obtain the total emittance[i],

$$\varepsilon_n = \sqrt{\varepsilon_{thermal}^2 + \varepsilon_{rf}^2 + \varepsilon_{sc}^2}\ . \qquad \text{Eqn. 1}$$

The rf emittance refers to a time-dependent focusing of the beam by the rf fields at the entrance and exit of each rf cavity. The rf focusing kick occurs at the ends of the cavities as a simple result of Maxwell's equations as described by the Panofsky-Wenzel theorem[ii]. This theorem simply states that the radial electric and azimuth magnetic fields are proportional to the z-derivative of the of the longitudinal field,

$E_r = -\frac{r}{2}\frac{\partial}{\partial z}E_z$, $cB_\theta = \frac{r}{2c}\frac{\partial}{\partial t}E_z$. Where the radial force is $F_r = e(E_r - \beta c B_\theta)$.

where E_z and E_r are the electric field components of a cylindrically symmetric rf cavity. The change in radial momentum is the integral of the force,

$$p_r = \frac{1}{mc}\int F_r dt = \frac{1}{mc^2}\int F_r \frac{dz}{\beta}. \qquad \text{Eqn. 2}$$

To a good approximation, the longitudinal rf field can be represented as,

$E_z = E(z)\cos kz \sin(\omega t + \phi_0)$, resulting in the following integrals for the radial momentum,

$$F_r = er\left\{-\frac{1}{2}\left(\frac{dE(z)}{dz}\right)\cos kz \sin(\omega t + \phi_0) - \frac{1}{2c}\frac{d}{dt}\left(E(z)\sin kz \cos(\omega t + \phi_0)\right) + \frac{\beta}{2}\frac{dE(z)}{dz}\sin kz \cos(\omega t + \phi_0)\right\}$$

Eqn. 3

The second term integral is zero since it is a total time derivative of an expression which is zero at the cathode (z=0) and beyond exit of the gun. The first and third terms are non-zero only where $dE(z)/dz$ is non-zero which is only at the gun exit. If we assume the impulse approximation, such that *E(z)* is constant inside the gun with a sharp cutoff at the cathode and gun exit, $E(z) = E_0\theta(z_f - z)$. Then the derivatives $dE(z)/dz$ becomes delta functions with the radial kick in terms of Kim's dimensionless momentum given to the beam only at the exit of the gun, where $z = z_f$

$$\Delta p_r = r\frac{eE_0}{2mc^2}\left[\beta \cos kz_f \sin(\omega t + \phi_0) - \sin kz_f \cos(\omega t + \phi_0)\right]$$

written as Kim's dimensionless momentum. Since the gun is 1 ½ cells long, $\sin kz_f = 0$ and $\cos kz_f = 1$. In addition at the exit iris of the gun the beam is relativistic, $\beta = 1$, and the radial momentum kick becomes,

$$\Delta p_r = \frac{eE_0 r}{2c}\sin(kz_f - \omega t_f - \phi_0) = \frac{eE_0 r}{2c}\sin\phi_e. \qquad \text{Eqn. 4}$$

Where the electron phase at the exit of the gun is $\phi_e = kz_f - \omega t_f - \phi_0$. Writing this in terms of the radial angle change gives the focal strength of the rf lens at the gun's exit,

$$\Delta p_r = \gamma mc\Delta r' \Rightarrow \Delta r' = r\frac{eE_0}{2\gamma mc^2}\sin\phi_e = -\frac{r}{f_{rf}} \qquad \text{Eqn. 5}$$

$$\therefore f_{rf} = -\frac{2\gamma mc^2}{eE_0\sin\phi_e}. \qquad \text{Eqn. 6}$$

Electrons at various longitudinal positions along the bunch length arriving at different phases at the gun exit will experience different kicks as illustrated in Figure 18, causing an increase in the projected emittance. Clearly the rf emittance is a minimum when the rf focal length is independent of ϕ_e. This occurs when

$$\frac{df_{rf}}{d\phi_e} = \frac{2\gamma mc^2}{eE_0}\frac{\cos\phi_e}{\sin^2\phi_e} = \frac{2\gamma mc^2}{eE_0}\frac{\cot\phi_e}{\sin\phi_e} \qquad \text{Eqn. 7}$$

is zero, which occurs when $\phi_e = \pi/2$.

Even with $\phi_e = \pi/2$ there is still an increase due to the curvature of the rf waveform. For this reason, RF guns typically are operated with bunch lengths no more than 10degrees of rf phase long. The emittance growth due to rf curvature can be greatly reduced by introducing a third harmonic of the fundamental rf frequency[iii].

The above derivation follows that of Kim[i] who also gives the rf emittance for a Gaussian beam with an root-mean-square width of σ_ϕ exiting the gun at $\phi_e = \pi/2$ which includes the effect of the curvature,

$$\varepsilon_x^{rf} = \frac{eE_0}{2mc^2}\frac{\langle x^2\rangle\sigma_\phi^2}{\sqrt{2}} \qquad \text{Eqn. 8}$$

Additional useful analytic formulae and their accuracy are discussed by Travier[iv].

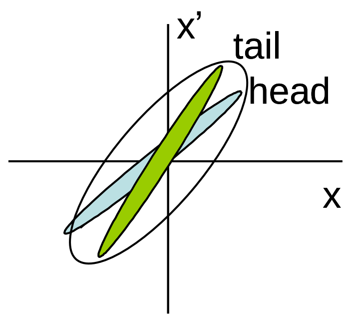

Figure 1

This radial field is significant in a high gradient rf gun as shown in Figure 2 where the longitudinal and transverse rf fields are shown for a gun with 100MV/m peak electric field on the cathode. In this case, the peak transverse field is 15MV/m or 15% of the longitudinal field, corresponding to a focal length of only 10cm for a beam with exit energy of 5MeV.

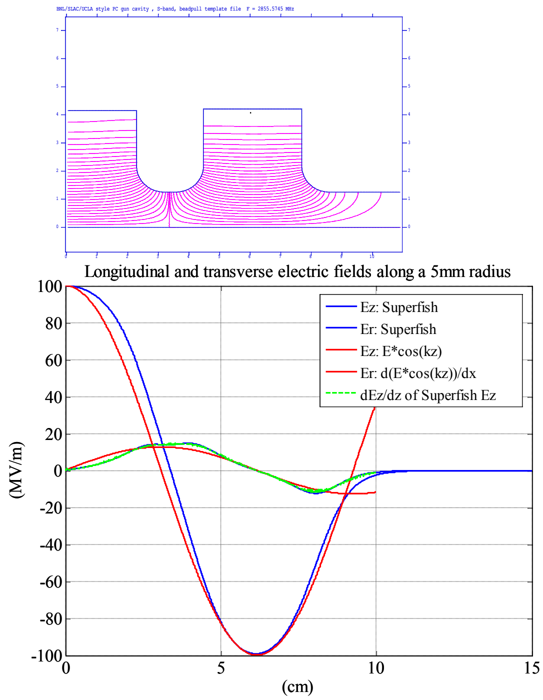

Figure 2

RF fields for the s-band gun computed[v] indicate strong radial fields in a rf gun operating at ~100MV/m.

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i. [K-J Kim, NIM 1989]
ii. Panofsky-Wenzel paper
iii. [Serafini, et al.]
iv. [Travier ref]
v. Superfish ref.